Mesa College – Math 141 - SAMPLES

In order to adequately prepare for the Math 141 (Precalculus) Challenge exam, the student should actually look at the Math 104 (Trig) and the Math 116 (College Algebra) samples. Only after you have thoroughly worked and re-worked those sample problems, should the student attempt the following, more advanced, examples.

Directions: **NO CALCULATOR.** Write neatly, show your work and steps. Label your work so it's easy to follow. **Answers without appropriate work will receive NO credit. For final answers, be sure to simplify all radicals and fractions.**

#1 – 6. Simplify completely.

1. $5^{\log_{25} 3}$
2. $10^{3+\log 4}$
3. $\sqrt{\dfrac{b^{1-n}}{b^{n-1}}}, \quad b>0$
4. $\sqrt[3]{\dfrac{125^n \bullet 5^{4n}}{25^{-n}}}$
5. $\dfrac{2^{-1}}{2^{-2}+2^{-3}}$
6. $\dfrac{x^{-2}-y^{-2}}{x^{-1}-y^{-1}}$

#7 - 10. Solve over the set of Complex numbers.

7. $ix^2-3x=2i$
8. $2^x+8\left(2^{-x}\right)=9$
9. $3^{2x+1}-10\left(3^x\right)+3=0$
10. $\dfrac{5^{x^2}}{\left(5^x\right)^2}=125$

#11 – 17. Simplify completely. Final answers should have only positive exponents.

11. $x^{\frac{-1}{2}}\left(x^{\frac{5}{2}}-2x^{\frac{3}{2}}\right)$
12. $\dfrac{2x^{\frac{1}{3}}\left(3x^{\frac{1}{3}}-4x^{\frac{4}{3}}\right)}{2x^{\frac{-1}{3}}}$
13. $\dfrac{\left(\sqrt[3]{4x}\right)^2}{\sqrt[6]{4x}}$
14. $(x-1)^{\frac{1}{2}}-x(x-1)^{\frac{-1}{2}}$
15. $\left(1+x^2\right)^{\frac{-3}{2}} - (1+x^2)^{\frac{-1}{2}}$
16. $\frac{4x}{3}(2x^2+3)^{\frac{-2}{3}}(x-5)^{\frac{2}{5}} + \frac{2}{5}(2x^2+3)^{\frac{1}{3}}(x-5)^{\frac{-3}{5}}$
17. $\dfrac{\left(\frac{1}{3}\right)(x-3)^{\frac{3}{4}}(2x+1)^{\frac{-2}{3}} - \frac{3}{4}(2x+1)^{\frac{1}{3}}(x-3)^{\frac{-1}{4}}}{(x-3)^{\frac{3}{2}}}$

#18 - 22. Solve each trigonometric equation. Use inverse notation if necessary.

18. $2\sin^2(2\theta)-1=0$, for $0^o \le \theta < 360^o$
19. $(\sin x)(\sin x-1)=\cos^2 x$, for $0\le x<2\pi$
20. $\sin(2x)=3\sin x$ for all real values of x.
21. $\cos x=2-2\cos^2(\frac{x}{2})$, for $0\le x<2\pi$
22. $3\csc^2 x+4\csc x=0$, for $0\le x<2\pi$.

#23 – 24. Evaluate, using exact real values. Assume principal values for inverse trig.

23. $\cos\left(Arctan\left(\frac{-5}{12}\right)\right)$ 24. $\sin\left(Cos^{-1}\left(\frac{3}{5}\right) + \frac{\pi}{3}\right)$

#25-26. Graph one period of each. Label at least one maximum and one minimum point, the y-intercept, one zero (if it exists) and the equations of all asymptotes, where applicable.

25. $y = 2\sin\frac{1}{2}(x) + 1$ 26. $y = \sec(3x - \pi)$

#27 – 31. Graph each. Label any x- and y-intercepts, and all asymptotes, where applicable.

27. $y = 2^x + 3$ 28. $y = e^{x+1}$ 29. $y = 3^{-x}$

30. $y = \log(x+2)$ 31. $y = \ln(x)$

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Solutions

#1. $\sqrt{3}$ #2. 4,000 #3. $\left|b^{1-n}\right|$ #4. 5^{3n} or 125^n #5. $\frac{4}{3}$

#6. $\frac{x+y}{xy}, y \neq x \neq 0$ #7. $\{-2i, -i\}$ #8. $\{3, 0\}$ #9. $\{1, -1\}$ #10. $\{3, -1\}$

#11. $x^2 - 2x$ #12. $3x - 4x^2$ #13. $2\sqrt{x}$ #14. $\frac{-1}{\sqrt{x-1}}$ or $\frac{-\sqrt{x-1}}{x-1}$

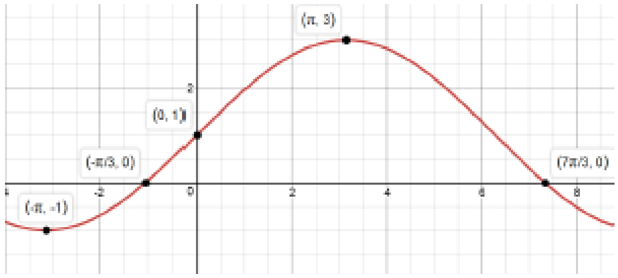
#15. $\frac{-x^2}{(x^2+1)^{\frac{3}{2}}}$ #16. $\frac{32x^2 - 100x + 18}{15(2x^2+3)^{\frac{2}{3}}(x-5)^{\frac{1}{5}}}$ #17. $\frac{-14x - 21}{12(2x+1)^{\frac{3}{4}}(x-3)^{\frac{2}{3}}}$

#18. $\{22.5^\circ,\ 202.5^\circ,\ 67.5^\circ,\ 247.5^\circ,\ 112.5^\circ,\ 292.5^\circ,\ 157.5^\circ,\ 337.5^\circ\}$

#19. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ #20. $x = 0, \pi$, but for <u>all</u> solutions: $\{x : x = k\pi, k \in Integers\}$ #21. $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

#22. $\left\{x \mid \pi+\sin^{-1}\left(\frac{3}{4}\right),\ 2\pi+\sin^{-1}\left(\frac{-3}{4}\right),\ 2\pi-\sin^{-1}\left(\frac{3}{4}\right)\right\}$, #23. $\frac{12}{13}$ #24. $\frac{4+3\sqrt{3}}{10}$

#25.



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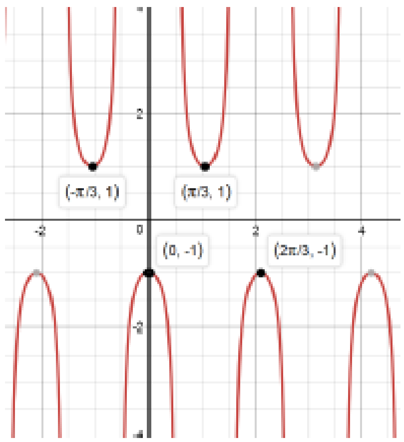

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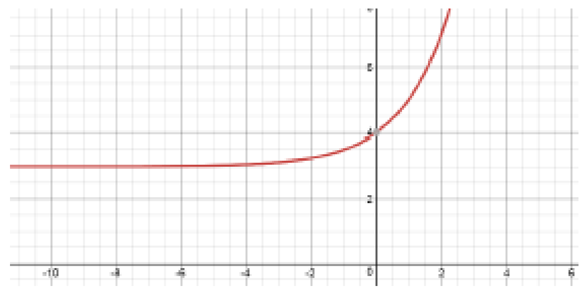

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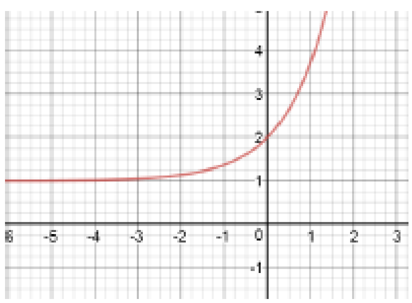

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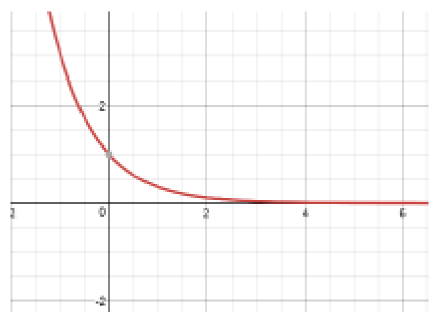

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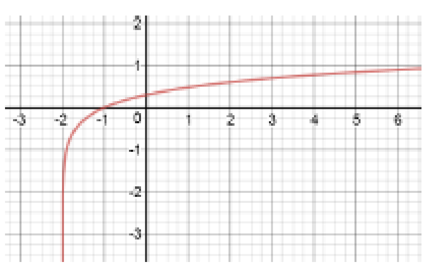
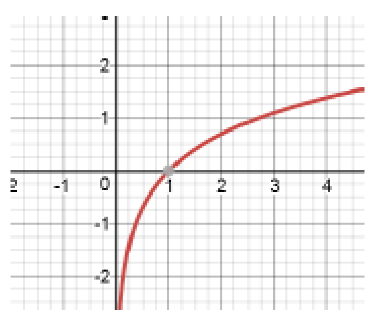


#31.



Mesa College – Math 104 (Trig) Challenge Exam SAMPLES

Directions: **NO CALCULATOR**. Write neatly and show your work and steps. Answers without appropriate work shown will receive little or NO credit. **Be sure to simplify all radicals and fractions.** Attach your neat and organized solution sheets behind this cover sheet. Make sure each solution is properly labeled.

#1 – 3. Use the given information to find the values of the remaining Trigonometric functions.

1. If $\sin\theta = -\frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$

2. If $\sin\theta = \frac{3}{5}$ and θ is in QII,

3. If $\cot\theta = -4$ and $270^o \le \theta < 360^o$

#4-6 . Find ALL values of x. Express your answers using radians.

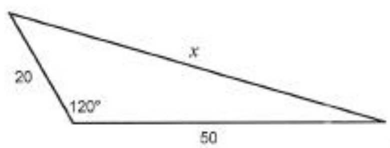
4. $3\csc x - 6 = 0.$

5. $\tan^2 x = -\tan x$

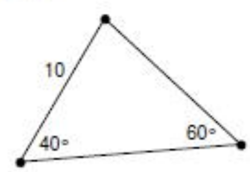
6. $2\sin^2 x - 5\sin x - 3 = 0$

7. Find the side length *x* in each of the triangles shown.

(7a)



(7b)



8. If $\cos x = 0.8$ and $\sin x = -0.6$, find :

(8a) $\sin(2x)$ (8b) $\cos(2x)$ (8c) in which quadrant is angle (2x) ?

9. Given that $\cos\alpha = \frac{2}{3}$ and $\sin\beta = -\frac{1}{4}$, where $\frac{-\pi}{2} \le \beta \le \frac{\pi}{2}$ and $0 \le \alpha \le \pi$ find :

(9a) $\sin(\alpha+\beta)$ (9b) $\cos(\alpha-\beta)$ (9c) $\cos\left(\frac{\alpha}{2}\right)$

(9d) in which quadrant is angle ($\alpha+\beta$)? (9e) in which quadrant is angle ($\alpha-\beta$)?

(9f) in which quadrant is angle $\left(\frac{\alpha}{2}\right)$?

10. Find the principal value of each:

10a) $Arc\cos\left(\frac{-\sqrt{3}}{2}\right)$ 10b) $\sin^{-1}\left(-\frac{1}{2}\right)$ 10c) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ 10d) $\operatorname{arc}\cot\left(\sqrt{3}\right)$

10e) $\cos\left(\arcsin\left(\sqrt{3}/2\right)\right)$ 10f) $\cos\left(\arctan(-1)\right)$ 10g) $\cot^{-1}\left(\sin\frac{3\pi}{2}\right)$

11. Find the <u>exact values</u> of each:

11a) $\sin\left(\frac{\pi}{4}\right)$ 11b) $\cos\left(30^o\right)$ 11c) $\csc\left(\frac{-3\pi}{4}\right)$ 11d) $\sec\left(-420^o\right)$

11e) $\cot\left(\frac{13\pi}{6}\right)$ 11f) $\dfrac{\sin\left(\frac{\pi}{6}\right)}{1+\cos\left(\frac{\pi}{6}\right)}$ 11g) $\cos\left(45^o\right)\cos\left(60^o\right)-\sin\left(45^o\right)\sin\left(60^o\right)$

12. Simplify each expression completely.

(12a) $\sin(75^\circ)\cos(15^\circ)-\cos(75)\sin(15)$ (12b) $\sin^2\left(\frac{3\pi}{8}\right)-\cos^2\left(\frac{3\pi}{8}\right)$ (12c) $\cos\left(\frac{5\pi}{8}\right)$

13. Find all solutions between $0\le\theta<360^o$ for each:

(13a) $\cos^2\theta-2\sin\theta=-2$ (13b) $\csc\left(\frac{x}{2}\right)=\sqrt{2}$ (13c) $\sin(2x)=\cos(x)$

14. Sketch at least one period of the graph of each. Label the <u>coordinates</u> of one maximum point <u>and</u> the <u>coordinates</u> of one minimum point.

(14a) $f(x)=-2\cos\left(x+\frac{\pi}{2}\right)+1$ (14b) $g(x)=5\sin 2\left(x-\frac{\pi}{4}\right)$ (14c) $h(x)=3+2\sec x$

15. **Simplify the expression** $\dfrac{\cot y-1}{1-\tan y}$, so that it matches one of the expressions below. SHOW STEPS.

(a) $\cos y$ (b) $\tan y$ (c) 0 (d) $\dfrac{\sec y}{\csc y}$ (e) $\cot y$

16. Simplify each expression.

(16a) $(\sec\beta-\tan\beta)(\sec\beta+\tan\beta)$ (16b) $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{1+\sin\theta}{\cos\theta}$

17. Complete the **trigonometric identity:** $\sec\theta-\cos\theta=w$, by selecting *w* from the list of expressions below.

(a) $\tan\theta\sin\theta$ (b) $\cot^2\theta\cos\theta$ (c) $\dfrac{1-\cos\theta\sin\theta}{\sin\theta}$ (d) $\dfrac{\sec\theta-1}{\sec\theta}$ (e) -1

18. SIMPLIFY each to a <u>single</u> trig expression involving no fractions.

(18a) $1+\dfrac{\tan^2 x}{1+\tan^2 x}$ (18b) $\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2-1$

19. If $A = 3+2i$ and $B = 5-i$, where $i = \sqrt{-1}$, find each:

(19a) A^2 (19b) AB (19c) $\dfrac{A}{B}$

20. Evaluate: $2i^2 - 5i^{75} + \dfrac{6}{i^{13}}$, where $i = \sqrt{-1}$.

21. Express each in standard form (i.e. $a \pm bi$)

(21a) $5\sqrt{2}\left(\cos\left(\frac{5\pi}{4}\right) - i\sin\left(\frac{5\pi}{4}\right)\right)$ (21b) $8cis30^o$

22. Convert each from Polar coordinates to Cartesian coordinates

(22a) $\left(2, 30^o\right)$ (22b) $\left(-6, \dfrac{4\pi}{3}\right)$

23. Convert each from Cartesian coordinates to Polar coordinates

(23a) $\left(3, \sqrt{3}\right)$ (23b) $(-5,-5)$

24. If $4+3i \simeq 5cis37^o$, find $(4+3i)^4$ in trigonometric form.

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Answers to Math 104 (Trig) SAMPLES

{ This is a DRAFT… If you find any errors, please contact me at: lafoster@sdccd.edu … Thanks. }

1. $\sin\theta = \dfrac{-1}{2}$, $\cos\theta = \dfrac{\sqrt{3}}{2}$, $\csc\theta = -2$, $\sec\theta = \dfrac{2\sqrt{3}}{3}$, $\tan\theta = \dfrac{-\sqrt{3}}{3}$, $\cot\theta = -\sqrt{3}$

2. $\sin\theta = \dfrac{3}{5}$, $\cos\theta = \dfrac{-4}{5}$, $\csc\theta = \dfrac{5}{3}$, $\sec\theta = \dfrac{-5}{4}$, $\tan\theta = \dfrac{-3}{4}$, $\cot\theta = \dfrac{-4}{3}$

3. $\sin\theta = \dfrac{-\sqrt{17}}{17}$, $\cos\theta = \dfrac{4\sqrt{17}}{17}$, $\csc\theta = -\sqrt{17}$, $\sec\theta = \dfrac{\sqrt{17}}{4}$, $\tan\theta = \dfrac{-1}{4}$, $\cot\theta = -4$

4. $\left\{x \mid x = \dfrac{\pi}{6} + 2k\pi \;\; or \;\; x = \dfrac{5\pi}{6} + 2k\pi\right\}$ 5. $\left\{x \mid x = k\pi \;\; or \;\; x = \dfrac{3\pi}{4} + k\pi\right\}$

6. $\left\{x \mid x = \dfrac{7\pi}{6} + 2k\pi \;\; or \;\; x = \dfrac{11\pi}{6} + 2k\pi\right\}$ 7a. $x = 10\sqrt{39}$ 7b. $x = \dfrac{20\sin 80^o}{\sqrt{3}}$

8a. $\frac{-24}{25} = -0.96$ 8b. $\frac{7}{25} = 0.28$ 8c. Q-IV 9a. $\frac{5\sqrt{3}-2}{12}$ 9b. $\frac{2\sqrt{15}-\sqrt{5}}{12}$

9c. $\frac{+\sqrt{30}}{6}$ 9d. 9e. 9f. all are in Q-I 10a. $\frac{5\pi}{6}$ 10b. $\frac{-\pi}{6}$ 10c. $\frac{\pi}{6}$ 10d. $\frac{\pi}{6}$

10e. $\frac{1}{2}$ 10f. $\frac{\sqrt{2}}{2}$ 10g. undefined. 11a. $\frac{\sqrt{2}}{2}$ 11b. $\frac{\sqrt{3}}{2}$

11c. $-\sqrt{2}$ 11d. 2 11e. $\sqrt{3}$ 11f. $2-\sqrt{3}$ 11g. $\frac{\sqrt{2}-\sqrt{6}}{4}$

12a. $\frac{\sqrt{3}}{2}$ 12b. $\frac{\sqrt{2}}{2}$ 12c. $\frac{\sqrt{2-\sqrt{2}}}{2}$ 13a. $\{90^o\}$

13b. $\{90^o, 270^o\}$ 13c. $\{90^o, 270^o, 30^o, 150^o\}$ 14. GRAPHS... see below

15. E 16a. 1 16b. $2\sec\theta$ 17. A

18a. $\cos^2 x$ 18b. $\sin x$ 19a. $5+12i$

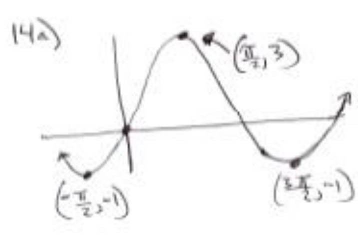
19b. $17+7i$ 19c. $\frac{1}{2}+\frac{1}{2}i$ 20. $-2-i$

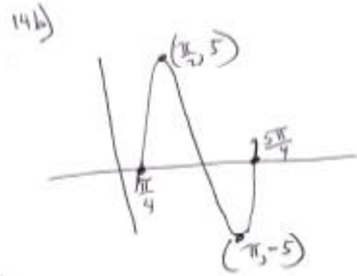
21a. $-5+5i$ 21b. $4\sqrt{3}+4i$ 22a. $(\sqrt{3},1)$ 22b. $(3,3\sqrt{3})$

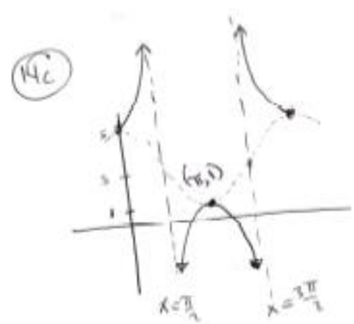
23a. $\left(2\sqrt{3},\frac{\pi}{6}\right)$

23b. $\left(5\sqrt{2},\frac{5\pi}{4}\right)$

24. $625cis(148^o)$







Mesa College – Math 116 - SAMPLES

Directions: **NO CALCULATOR.** Write neatly, show your work and steps. Label your work so it's easy to follow. Answers without appropriate work will receive NO credit. **For final answers, be sure to simplify all radicals and fractions.**

#1. Find the domain of each function. Express your answer in interval notation.

1a) $f(x)=\dfrac{\sqrt{4x-3}}{x^2-4}$ 1b) $g(x)=\dfrac{\sqrt{3-5x}}{4x^2-25}$ 1c) $h(x)=\dfrac{x^2-3x-10}{\sqrt{3x-8}}$

#2. Find $\dfrac{f(a+h)-f(a)}{h}$. Completely simplify your result, where:

2a) $f(x)=-x^2+x+5$ 2b) $f(x)=2x^2-5x$ 2c) $f(x)=\dfrac{2}{x}$

#3. Find $(f\circ g)(x)$, and specify the domain of $f\circ g$ using interval notation, where:

3a) $f(x)=\dfrac{x}{3x+2}$ and $g(x)=\dfrac{2}{x}$ 3b) $f(x)=\dfrac{2x}{x-4}$ *and* $g(x)=\dfrac{2}{x+1}$ 3c) $f(x)=g(x)=\dfrac{x}{x+1}$

#4. Find all x such that $f(x)\geq 0$. Write your answer using interval notation, where:

4a) $f(x)=-x^4+x^3+2x^2$. 4b) $f(x)=2x^3-x^2-8x+4$ 4c) $f(x)=\dfrac{2x-4}{x^2-9}$

#5. Find the equations of all vertical and horizontal asymptotes for :

5a) $f(x)=\dfrac{x^2-2x+1}{x^3-x^2+x-1}$ 5b) $g(x)=\dfrac{2x^2-x-1}{4x^2-x-3}$ 5c) $h(x)=\dfrac{5x^3+3x}{4x^3-9x}$

#6. Express as a single logarithm. Assume all values are properly defined

6a) $\log\dfrac{x^2}{y^3}+4\log y-6\log\sqrt{xy}$ 6b) $\log_b\dfrac{b}{\sqrt{x}}+\log_b\sqrt{bx}$

#7. Solve:

7a) $\log_3 3x=\log_3 x+\log_3(4-x)$. 7b) $\log_3(x)-\log_3(x+1)=3$ 7c) $\log_x 25=\dfrac{2}{3}$

7d) $\log_3 x+\log_3(x+6)=2$ 7e) $27^{2x-5}=9\left(3^{7x+1}\right)$ 7f) $\log_2(3x-1)=5$

7g) $23=e^{-0.4x}$ 7h) $\log_x 5=3$

#8. Suppose $A=\begin{bmatrix}1 & 2\\ 4 & -3\end{bmatrix}$, $B=\begin{bmatrix}-1\\ 5\end{bmatrix}$, and $C=\begin{bmatrix}0 & 3\\ -2 & 1\end{bmatrix}$. Find the following:

8a) AB   8b) 3A - 2C   8c) CB   8d) BC   8e) A^{-1}

#9. Use augmented matrices and Gaussian elimination to solve:

9a) $\begin{cases}3x+y-2z=1\\ 2x+3y-z=2\\ x-2y+2z=-10\end{cases}$ 9b) $\begin{cases}x-2y+3z=9\\ -x+3y=-4\\ 2x-5y+5z=17\end{cases}$ 9c) $\begin{cases}2x+3y=-2\\ 5y-2z=4\\ 4x+3z=-7\end{cases}$

#10. Maximize $P=10x+7y$, given the constraints $0\le x\le 60,\ 0\le y\le 45,$ and $5x+6y\le 420$. Sketch the graph of the constraint inequalities. Label the critical vertices on the graph.

#11. An accountant prepares tax returns for individuals and for small businesses. On average, each individual return requires 3 hours of her time, and 1 hour of computer time. Each business requires 4 hours of her time and 2 hours of computer time. Because of other business considerations, her time is limited to 240 hours, and the computer time is limited to 100 hours. If she earns a profit of $80 per each individual return, and a profit of $150 on each business return, how many returns of each type should she prepare to maximize her profit?

#12. SOLVE each over the complex number system.

12a) $x^4-5x^2=36$ 12b) $x^{\frac{-4}{3}}-5x^{\frac{-2}{3}}+4=0$

12c) $\sqrt{2x^2-7}=x+3$ 12d) $2\left(x+\frac{1}{x}\right)^2-7\left(x+\frac{1}{x}\right)+5=0$

#13. Sketch the graph of each polynomial function labeling the zeros and the y-intercepts for each.

13a) $y=2x^3-5x^2-4x+3$ 13b) $y=x^4-3x^2+2x$

#14. Let F(x) be a polynomial function with rational coefficients. Solve each, over the complex numbers, if necessary, using the given hints.

14a) $x^3+2x^2-23x-60=0$, where 5 is a root.

14b) $2x^4-3x^3-7x^2-8x+6=0$, where $\frac{1}{2}$ is a root

#15a) Write a third degree polynomial equation with <u>rational</u> coefficients, that has $2\sqrt{5}$ and 1 as its roots

#15b) Write a fourth degree polynomial equation with <u>integer</u> coefficients that has $3i$ (where $i=\sqrt{-1}$) and -1 as roots, where -1 is a 'double' root. (also called: -1 has a multiplicity of two.)

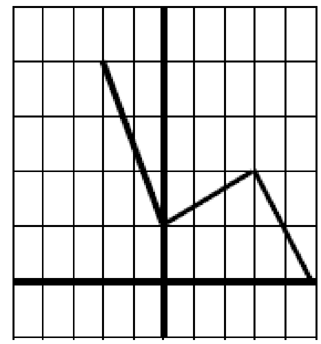
#16. The below graph represents part of a function y = f(x). Sketch each:

16a) y = f(x+2) 16b) $y = f(2x)$

16c) y = $f^{-1}(x)$ 16d) $y = f(x) - 2$

16e) the reflection of f(x) over the y-axis

16f) the reflection of f(x) over the x-axis



#17. Suppose f(x) is a continuous, one-to-one function, find:

17a) $f(-2)$, if $f(2) = 5$, and f(x) is an 'even' function

17b) $f(-2)$, if $f(2) = 5$, and f(x) is an 'odd' function

#18. Sketch the graph of each piece-wise function. Label the coordinates of each vertex and/or each point of discontinuity.

18a) $y = \begin{cases} |x|, \text{ if } -2 \le x < 1 \\ 2x - 1, \text{ if } x \ge 1 \end{cases}$ 18b) $y = \begin{cases} 1, \text{ if } x < -2 \\ x + 1, \text{ if } -2 \le x \le 3 \\ 2, \text{ if } x > 3 \end{cases}$

#19. Resolve each into its <u>partial fractions</u>.

19a) $\dfrac{x+2}{2x^2 - 7x - 15}$ 19b) $\dfrac{2x^2 + 7x + 23}{(x-1)(x+3)^2}$

#20. A new drug is injected into a patient. The number of milligrams remaining in the patient's bloodstream '*t*' hours is modeled by: $D(t) = 60e^{-0.2t}$. (a) How many milligrams were initially injected? (b) How many milligrams of the drug remain after 5 hours? (c) When will there be one-forth of the initial dosage in the patient's bloodstream?

#21. Students in a math class took a final exam. As part of a study, they were tested each month thereafter to determine how much they remembered. The following formula was derived: $S(t) = 77.4 - 20\log(t+1),\ t \ge 0.$ Where S(t) represents the score (as a percent), and '*t*' represents months. (a) What was the average initial score on the exam? (b) What was the average score after 9 months?

ANSWERS to Math 116 Challenge Exam – SAMPLES

Disclaimer: A few of these answers may (will) be wrong! There's a myriad of reasons why: I'm stooped, my fingers are fat, I'm dyslexic, I'm tired, the summer sun has baked my brain. Please, email me your suggested solution(s), I'll double check it, and get back to you. Thanks. Larry Foster: lafoster@sdccd.edu

NOTE: I have NOT yet included the GRAPHS … someday.

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1a) Domain = $\left[\frac{3}{4},2\right)\cup(2,\infty)$ 1b) Domain = $\left(-\infty,\frac{-5}{2}\right)\cup\left(\frac{-5}{2},\frac{3}{5}\right]$ 1c) Domain = $\left(\frac{8}{3},\infty\right)$

2a) -2a + 1 – h 2b) 4a – 5 + 2h 2c) $\dfrac{-2}{a^2+ah}$

3a) $f\circ g=\dfrac{1}{x+3}$, domain = $(-\infty,-3)\cup(-3,0)\cup(0,\infty)$

3b) $f\circ g=\dfrac{-2}{2x+1}$, domain = $(-\infty,-1)\cup\left(-1,\frac{-1}{2}\right)\cup\left(\frac{-1}{2},\infty\right)$

3c) $f\circ g=\dfrac{x}{2x+1}$, domain = $(-\infty,-1)\cup\left(-1,\frac{-1}{2}\right)\cup\left(\frac{-1}{2},\infty\right)$

4a) $f(x)>0, when\ x\in[-1,2]$ 4b) $f(x)>0, when\ x\in\left[-2,\frac{1}{2}\right]\cup[2,\infty)$

4c) $f(x)>0, when\ x\in[-3,2]\cup(3,\infty)$

5a) $f(x)=\dfrac{x-1}{x^2+1}, x\neq 1$, vertical Asymptote: none, horizontal Asymptote: $y=0$,
Point of Discontinuity (aka: 'hole'): $(0,-1)$

5b) $g(x)=\dfrac{2x+1}{4x+3}, x\neq 1$, vert: $x=\frac{-3}{4}$, horiz: $y=\frac{1}{2}$, hole @ $\left(1,\frac{1}{3}\right)$

5c) $h(x)=\dfrac{5x+3}{(2x+3)(2x-3)}, x\neq 0$, vert: $x=\pm\frac{3}{2}$, horiz: $y=0$, hole @ $\left(0,\frac{-1}{3}\right)$

6a) $-\log(xy)$ 6b) $1\frac{1}{2}$

7a) $\{1\}$, 0 rejects 7b) $\{\ \}$, $\dfrac{-27}{26}$ rejects 7c) $\{125\}$ 7d) $\{3\}$, -9 rejects

7e) $\{-18\}$ 7f) $\{11\}$ 7g) $\left\{\dfrac{\ln 23}{-0.04}\right\}$ this is best possible w/o calculator 7h) $\left\{\sqrt[3]{5}\right\}$

8a) $\begin{bmatrix}-19\\-19\end{bmatrix}$ 8b) $\begin{bmatrix}3 & 0\\16 & -11\end{bmatrix}$ 8c) $\begin{bmatrix}-10\\7\end{bmatrix}$ 8d) not defined 8e) $\begin{bmatrix}\frac{3}{11} & \frac{2}{11}\\\frac{4}{11} & \frac{-1}{11}\end{bmatrix}$

9) after a LOT of work: 9a) $\{(-2, 1, -3)\}$ 9b) $\{(1, -1, 2)\}$ 9c) $\{(-4, 2, 3)\}$

Remember: the parenthesis inside of the solution set is mandatory!

10) See below for sketch. corners: (0,0), (0,45), (60,0), 30,45), (60,20) The max value of P occurs when x = 60 and y = 20.

11) Let x = # of individual returns, y = # of business returns Objective function: P = 80x + 150y

Constraints: $\begin{cases} x \geq 0 \\ y \geq 0 \\ 3x + 4y \leq 240 \\ x + 2y \leq 100 \end{cases}$ corners: (0,0), (0, 50), (40,30), 80,0)

Max profit occurs @ (40,30) she should prepare 40 individual returns, and 30 business returns.

12a) $\{\pm 3, \pm 2i\}$ 12b) $\{\frac{1}{8}, 1\}$ 12c) $\{8, -2\}$ 12d) $\{2, \frac{1}{2}, \frac{1}{2} \pm \frac{i\sqrt{3}}{2}\}$

#13 See below for sketches. Using a combination of: Rational Root theorem, synthetic division, factoring, and quadratic formula:

13a) $y = (x+1)(2x-1)(x-3)$ zeros: $-1, 3, \frac{1}{2}$, y-intercept: $(0, 3)$ + check out it's 'extreme' behavior

13b) $y = x(x-1)^2(x+2)$, zeros: 0, -2, 1 (multiplicity of 2), y-int: (0,0)

#14 – see 'hints' from #13

14a) (x – 5)(x + 3)(x + 4) = 0 $\Rightarrow \{5, -3, 4\}$

14b) $(x - \frac{1}{2})(x-3)(x^2 + 2x + 2) = 0 \Rightarrow \{\frac{1}{2}, 3, -1 \pm i\}$

15a) remember with rational coefficients, each irrational and complex root must include its conjugate.

$$(x - 2\sqrt{5})(x + 2\sqrt{5})(x - 1) = 0 \Rightarrow x^3 - x^2 - 20x + 20 = 0$$

15b) $(x - 3i)(x + 3i)(x+1)(x+1) = 0 \Rightarrow x^4 + 2x^3 + 10x^2 + 18x + 1 = 0$

#16 graphs later. Each corner of the shape should be modified as indicated below. Then, connect the dots.

16a) shift entire graph left 2 units. (subtract 2 from x-values, keep y-values the same)

16b) (divide x-values by 2, keep y-values the same)

16c) swap all x $\leftrightarrow$ y that is: (-2,4) $\rightarrow$ (4, -2)

(0,1) $\rightarrow$ (1,0)

(3,2) $\rightarrow$ (2,3)

(5,0) $\to$ (0,5)

16d) shift entire graph <u>down</u> 2 units: (keep x-value the same, subtract 2 from y-values)

16e) (change all x-values to their opposite, keep y-value the same)

16f) (keep x-value same, change y-values to their <u>opposite)</u>

17a) $f(-2)=5$ 17b) $f(-2)=-5$

#18 – see below for sketches (someday)

19a) $\frac{-1}{13(2x+3)}+\frac{7}{13(x-5)}$ 19b) $\frac{2}{x-1}+\frac{-5}{(x+3)^2}+\frac{0}{x+3}$ of course you don't need this last fraction

20a) 60 mg 20b) $60e^{-1}$ mg, or $\frac{60}{e}$ mg 20c) $\frac{\ln(\frac{1}{4})}{-0.2}$ hours, or $\frac{\ln(4)}{0.2}$ hours.

21a) The initial average score was 77.4% 21b) After 9 months, the resulting average score was 57.4%